This PhD dissertation is devoted to the study of ergodic properties of Markov processes corresponding to systems of interval increasing homeomorphisms with probabilities. These systems in general arise in connection with fractals and partially hyperbolic dynamical systems. Our objects of interest appeared to be important due to relations to Kan's example of a diffeomorphism possessing two attractors with intermingled basins. This is briefly explained in Chapter 1.

Chapter 2 describes basic facts on the behaviour of the Markov processes under consideration. The dynamics depends on the values of, so called, the average Lyapunov exponents at 0 and 1. It is proved that if both exponents are positive, then there exists a stationary distribution μ with $\mu((0,1))=1$. In that case systems appear to be synchronizing, i.e. the distance between corresponding trajectories starting from two arbitrary points tends to 0 almost surely.

In Chapter 3 it is proved that the average distance between two trajectories is diminishing exponentially fast provided the system is consisted of C^2 diffeomorphisms. The proof strongly relies on certain version of the Baxendale theorem proved by Gharaei and Homburg, which says that the volume Lyapunov exponent of the system is negative. The exponential convergence allows us to show the classical probability limit theorems for the stochastic processes under consideration. The method is based on solving the Poisson equation and the Gordin method.

In the general case it is unknown whether the average distance between two trajectories is diminishing exponentially fast. Nevertheless it is possible to exploit certain result of Dominique Malicet from 2014 to estimate the average distance and show the classical limit theorems. Here the method is again based on martingale approximation and uses the Maxwell-Woodroofe criterion. This is the content of Chapter 4.

Chapter 5 is devoted to the study of very specific systems of homeomorphisms with place-dependent probabilities, called Alseda-Misiurewicz systems. All known methods of proving ergodicity and stability of iterated function systems with place-dependent probabilities rely on the contractivity in average, which for our system is not satisfied. Nevertheless we demonstrate that these properties hold.